

## Consistent boundary conditions for new massive gravity in $AdS_3$

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## Consistent boundary conditions for new massive gravity in $AdS_3$

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**ABSTRACT:** In this note we study the boundary conditions for the new massive gravity theory in asymptotically  $AdS_3$  spacetime. We find that the Brown-Henneaux boundary conditions are consistent with the new massive gravity for all any value of the mass parameter. At the critical point where the central charge vanishes, the conserved charges vanish, too. This provides further evidence that the theory may be trivial at the critical point under Brown-Henneaux boundary conditions. The log boundary conditions are also examined and we find that we can have three kinds of log boundary conditions for this new massive gravity theory, each of which could be consistent at the critical point, while for other value of the mass parameter, the log gravity boundary condition is not consistent.

**KEYWORDS:** AdS-CFT Correspondence, Models of Quantum Gravity

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The basic setup</b>	<b>2</b>
2.1	The new massive gravity theory	3
2.2	Conserved charges	4
<b>3</b>	<b>Brown-Henneaux boundary condition</b>	<b>6</b>
<b>4</b>	<b>Log boundary conditions</b>	<b>7</b>
4.1	Left moving relaxation	8
4.2	Right moving relaxation	9
4.3	Both modes relaxation	11
<b>5</b>	<b>Conclusion and discussion</b>	<b>12</b>

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## 1 Introduction

Recently in [1] a new kind of massive gravity has been discovered in three dimensions. In this new massive gravity, higher derivative terms are added to the Einstein Hilbert action and unlike in topological massive gravity, parity is preserved in this new massive gravity. This new massive gravity is equivalent to the Pauli-Fierz action for a massive spin-2 field at the linearized level in asymptotically Minkowski spacetime. In [2, 3], the unitarity of this new massive gravity and the new massive gravity with a Pauli-Fierz mass term was examined. Warped AdS black hole solutions for this new massive gravity with a negative cosmological constant have been found in [4].

In [5], the linearized gravitational excitations of this new massive gravity around asymptotically  $AdS_3$  spacetime has been studied. There are four branches of highest weight graviton solutions satisfying the Brown-Henneaux boundary conditions in this theory: the left and right moving massless gravitons and the left and right moving massive gravitons. It was found that there is also a critical point for the mass parameter at which massive gravitons become massless as in topological massive gravity [6, 7] in  $AdS_3$  [8] (other interesting discussions on topological massive gravity theory could be found in [9–26]). At the critical point, the leftmoving and rightmoving central charges are both zero and the energy of all branches of highest weight gravitons vanish under the Brown-Henneaux boundary conditions. It was conjectured that the new massive gravity may be trivial at the critical point under Brown-Henneaux boundary conditions. However, the consistency of the Brown-Henneaux boundary conditions with the new massive gravity in asymptotically  $AdS_3$  was

not shown in [5]. In this note we will first study the consistency of the Brown-Henneaux boundary conditions with the new massive gravity in asymptotically  $AdS_3$ . We find that Brown-Henneaux boundary conditions are consistent with the theory for any value of the mass parameter. At the critical point, both the left moving and right moving conserved charges are zero. This provides further evidence that the new massive gravity in  $AdS_3$  may be trivial at the critical point under Brown-Henneaux boundary conditions.

As is shown in [11], at the chiral point of topological massive gravity theory there can be a new kind of solution which does not obey Brown-Henneaux boundary conditions. This kind of solution appears at the chiral point because the left moving massless graviton and the left moving massive graviton degenerate at the point. To include this new solution to the theory, the boundary conditions should be relaxed to the log boundary conditions [22, 25, 26], and under this kind of log boundary conditions in topological gravity in  $AdS_3$ , the left moving conserved charge is no longer zero and the theory is not chiral at the chiral point. In this note, we show that the new kind of solution also exists in the new massive gravity. We can also take the log boundary conditions to include the new solutions. We find that under the log boundary conditions, the conserved charges are divergent for general value of the mass parameter while at the critical point, the log boundary condition is consistent with the theory.

Different from in Chiral gravity, because in this theory, the right moving massless and massive gravitons also degenerate at the critical point as well as the left moving modes, we can have two new solutions. We can also relax the boundary conditions to the log boundary condition in this theory to include the new solutions. Depending on which solutions we want to include, we can have three kinds of log boundary conditions: one for the left moving new solution, one for the right moving and one for both modes. In the first two cases, after imposing the log boundary conditions we can only get nonzero conserved charge for one of the two modes and the conserved charge for the other mode is still zero. This can be viewed as a new kind of log chiral gravity, which is realized by imposing different boundary conditions for the left moving and right moving modes respectively. In the last case, we can have nonzero conserved charges for both modes.

The structure of our work is as follows. In section 2 we will review the new massive gravity in asymptotically  $AdS_3$  and write out the formula for the calculation of conserved charges in this theory. In section 3 we examine the consistency of the Brown-Henneaux boundary conditions. In section 4 we study the log boundary conditions. Section 5 is devoted to conclusions and discussions.

## 2 The basic setup

In this section we will first review the new massive gravity theory [1, 5] with a negative cosmological constant and then derive the useful formulae for calculating the conserved charges with given boundary conditions.

## 2.1 The new massive gravity theory

The action of the new massive gravity theory can be written as<sup>1</sup> [1]

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ R - 2\lambda - \frac{1}{m^2} K \right], \quad (2.1)$$

where

$$K = R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2, \quad (2.2)$$

$m$  is the mass parameter of this massive gravity and  $\lambda$  is a constant which is different from the cosmological constant. The equation of motion of this action is

$$G_{\mu\nu} + \lambda g_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} = 0 \quad (2.3)$$

where

$$K_{\mu\nu} = -\frac{1}{2} \nabla^2 R g_{\mu\nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R + 2 \nabla^2 R_{\mu\nu} + 4 R_{\mu\alpha\nu\beta} R^{\alpha\beta} - \frac{3}{2} R R_{\mu\nu} - R_{\alpha\beta} R^{\alpha\beta} g_{\mu\nu} + \frac{3}{8} R^2 g_{\mu\nu}. \quad (2.4)$$

One special feature of this choice of  $K$  is that  $g^{\mu\nu} K_{\mu\nu} = K$ .

After introducing a non-zero  $\lambda$ , the new massive gravity theory could have an  $AdS_3$  solution

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = \ell^2 (-\cosh^2 \rho d\tau^2 + \sinh^2 \rho d\phi^2 + d\rho^2), \quad (2.5)$$

and the  $\lambda$  in the action should be related to the cosmological constant  $\Lambda$  and the mass parameter by

$$m^2 = \frac{\Lambda^2}{4(-\lambda + \Lambda)}, \quad (2.6)$$

and

$$\Lambda = -1/\ell^2. \quad (2.7)$$

It would be useful to introduce the light-cone coordinates  $\tau^\pm = \tau \pm \phi$ , then the  $AdS_3$  spacetime (2.5) could be written as

$$ds^2 = \frac{\ell^2}{4} (-d\tau^{+2} - 2 \cosh 2\rho d\tau^+ d\tau^- - d\tau^{-2} + 4d\rho^2). \quad (2.8)$$

For convenience we define

$$\mathcal{G}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu}, \quad (2.9)$$

and by expanding  $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$  around  $AdS_3$ , we could obtain the equation of motion for the linearized excitations  $h_{\mu\nu}$  as

$$(2m^2 + 5\Lambda) \mathcal{G}_{\mu\nu}^{(1)} - \frac{1}{2} (\bar{g}_{\mu\nu} \bar{\nabla}^2 - \bar{\nabla}_\mu \bar{\nabla}_\nu + 2\Lambda \bar{g}_{\mu\nu}) R^{(1)} - 2(\bar{\nabla}^2 \mathcal{G}_{\mu\nu}^{(1)} - \Lambda \bar{g}_{\mu\nu} R^{(1)}) = 0, \quad (2.10)$$

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<sup>1</sup> We take the metric signature  $(-,+,+)$  and follow the notation and conventions of MTW [27]. We assume  $m^2 > 0$  and  $G$  is the three dimensional Newton constant which is positive here.

where

$$R_{\mu\nu}^{(1)} = \frac{1}{2}(-\bar{\nabla}^2 h_{\mu\nu} - \bar{\nabla}_\mu \bar{\nabla}_\nu h + \bar{\nabla}^\sigma \bar{\nabla}_\nu h_{\sigma\mu} + \bar{\nabla}^\sigma \bar{\nabla}_\mu h_{\sigma\nu}), \quad (2.11)$$

$$R^{(1)} \equiv (R_{\mu\nu} g^{\mu\nu})^{(1)} = -\bar{\nabla}^2 h + \bar{\nabla}_\mu \bar{\nabla}_\nu h^{\mu\nu} - 2\Lambda h, \quad (2.12)$$

$$\mathcal{G}_{\mu\nu}^{(1)} = R_{\mu\nu}^{(1)} - \frac{1}{2}\bar{g}_{\mu\nu}R^{(1)} - 2\Lambda h_{\mu\nu}. \quad (2.13)$$

After gauge fixing, we can solve the equation of motion (2.10) and obtain two sets of solutions. One set is the left moving and right moving highest weight massless gravitons with weights (2, 0) and (0, 2) respectively. The other set is the  $(\frac{6+\sqrt{2+4m^2\ell^2}}{4}, \frac{-2+\sqrt{2+4m^2\ell^2}}{4})$  and  $(\frac{-2+\sqrt{2+4m^2\ell^2}}{4}, \frac{6+\sqrt{2+4m^2\ell^2}}{4})$  highest weight massive gravitons. Both the non-negativeness of the central charge and the non-negativeness of the mass of gravitons demand that  $m^2\ell^2 \geq 1/2$ .

Very analogous to topological massive gravity theory in  $AdS_3$ , interesting things happen at the special point  $m^2\ell^2 = 1/2$ . The massive gravitons become massless at this point and both the central charges and the energy of all branches of gravitons become zero. This implies that the theory may be trivial at the special point under Brown-Henneaux boundary conditions. In the next section we will show that Brown-Henneaux boundary conditions are consistent with this theory and at the critical point  $m^2\ell^2 = 1/2$  all conserved charges vanish.

## 2.2 Conserved charges

In this subsection we will give the basic formulae to calculate the conserved charges using the covariant formalism [26, 32–34] (see also [35–42]) for this new massive gravity.

The covariant energy momentum tensor for the linearized gravitational excitations of this new massive gravity theory can be identified as

$$32\pi m^2 G T_{\mu\nu} = (2m^2 + 5\Lambda)\mathcal{G}_{\mu\nu}^{(1)} - \frac{1}{2}(\bar{g}_{\mu\nu}\bar{\nabla}^2 - \bar{\nabla}_\mu \bar{\nabla}_\nu + 2\Lambda\bar{g}_{\mu\nu})R^{(1)} - 2(\bar{\nabla}^2\mathcal{G}_{\mu\nu}^{(1)} - \Lambda\bar{g}_{\mu\nu}R^{(1)}). \quad (2.14)$$

It can be checked that the conservation of this energy momentum tensor  $\bar{\nabla}^\mu T_{\mu\nu} = 0$  could be obtained from the following equations:

$$\begin{aligned} \bar{\nabla}^\mu \mathcal{G}_{\mu\nu}^{(1)} &= 0 \\ \bar{\nabla}^\mu (\bar{g}_{\mu\nu}\bar{\nabla}^2 - \bar{\nabla}_\mu \bar{\nabla}_\nu + 2\Lambda\bar{g}_{\mu\nu})R^{(1)} &= 0 \\ \bar{\nabla}^\mu (\bar{\nabla}^2\mathcal{G}_{\mu\nu}^{(1)} - \Lambda\bar{g}_{\mu\nu}R^{(1)}) &= 0. \end{aligned} \quad (2.15)$$

It's shown in [43, 44] that when the background spacetime admits a Killing vector  $\xi_\mu$ , the current

$$\mathcal{K}^\mu = 16\pi G \xi_\nu T^{\mu\nu} \quad (2.16)$$

is covariantly conserved  $\bar{\nabla}_\mu \mathcal{K}^\mu = 0$ . Then there exists an antisymmetric two form tensor  $\mathcal{F}^{\mu\nu}$  such that

$$\mathcal{K}^\mu = 16\pi G \bar{\nabla}_\nu \mathcal{F}^{\mu\nu} \quad (2.17)$$

and the corresponding charge could be written as a surface integral as

$$Q(\xi) = -\frac{1}{8\pi G} \int_M \sqrt{-\bar{g}} \mathcal{K}^0 = -\frac{1}{8\pi G} \int_{\partial M} dS_i \sqrt{-\bar{g}} \mathcal{F}^{0i}, \quad (2.18)$$

where  $\partial M$  is the boundary of a spacelike surface  $M$ . We have chosen  $M$  as constant time surface here and the expression is under the coordinate system of (2.5).

We can rewrite each term in the expression of the energy momentum tensor (2.14) as a total covariant derivative term [43, 44] using the definition and the following properties of Killing vectors:

$$\bar{\nabla}_\mu \xi^\mu = 0, \quad \bar{\nabla}_\sigma \bar{\nabla}^\mu \xi^\sigma = 2\Lambda \xi^\mu, \quad \bar{\nabla}^2 \xi_\sigma = -2\Lambda \xi_\sigma, \quad (2.19)$$

to be

$$2\xi_\nu \mathcal{G}^{(1)\mu\nu} = \bar{\nabla}_\sigma \left\{ \xi_\nu \bar{\nabla}^\mu h^{\sigma\nu} - \xi_\nu \bar{\nabla}^\sigma h^{\mu\nu} + \xi^\mu \bar{\nabla}^\sigma h - \xi^\sigma \bar{\nabla}^\mu h \right. \\ \left. + h^{\mu\nu} \bar{\nabla}^\sigma \xi_\nu - h^{\sigma\nu} \bar{\nabla}^\mu \xi_\nu + \xi^\sigma \bar{\nabla}_\nu h^{\mu\nu} - \xi^\mu \bar{\nabla}_\nu h^{\sigma\nu} + h \bar{\nabla}^\mu \xi^\sigma \right\} \quad (2.20)$$

$$\xi_\nu \left( \bar{g}^{\mu\nu} \bar{\nabla}^2 - \bar{\nabla}^\mu \bar{\nabla}^\nu + 2\Lambda g^{\mu\nu} \right) R^{(1)} = \bar{\nabla}_\sigma \left\{ \xi^\mu \bar{\nabla}^\sigma R^{(1)} + R^{(1)} \bar{\nabla}^\mu \xi^\sigma - \xi^\sigma \bar{\nabla}^\mu R^{(1)} \right\} \quad (2.21)$$

$$\xi_\nu \left( \bar{\nabla}^2 \mathcal{G}^{(1)\mu\nu} - \Lambda \bar{g}^{\mu\nu} R^{(1)} \right) = \bar{\nabla}_\sigma \left\{ \xi_\nu \bar{\nabla}^\sigma \mathcal{G}^{(1)\mu\nu} - \xi_\nu \bar{\nabla}^\mu \mathcal{G}^{(1)\sigma\nu} - \mathcal{G}^{(1)\mu\nu} \bar{\nabla}^\sigma \xi_\nu \right. \\ \left. + \mathcal{G}^{(1)\sigma\nu} \bar{\nabla}^\mu \xi_\nu \right\} + 2\Lambda \xi_\nu \mathcal{G}^{(1)\mu\nu}. \quad (2.22)$$

Thus we could write

$$\xi_\nu T^{\mu\nu} = \bar{\nabla}_\sigma \mathcal{F}^{\mu\sigma}, \quad (2.23)$$

where

$$\mathcal{F}^{\mu\sigma} = \left( 1 - \frac{1}{2m^2 \ell^2} \right) \frac{1}{2} \left\{ \xi_\nu \bar{\nabla}^\mu h^{\sigma\nu} - \xi_\nu \bar{\nabla}^\sigma h^{\mu\nu} + \xi^\mu \bar{\nabla}^\sigma h - \xi^\sigma \bar{\nabla}^\mu h \right. \\ \left. + h^{\mu\nu} \bar{\nabla}^\sigma \xi_\nu - h^{\sigma\nu} \bar{\nabla}^\mu \xi_\nu + \xi^\sigma \bar{\nabla}_\nu h^{\mu\nu} - \xi^\mu \bar{\nabla}_\nu h^{\sigma\nu} + h \bar{\nabla}^\mu \xi^\sigma \right\} \\ - \frac{1}{4m^2} \left\{ \xi^\mu \bar{\nabla}^\sigma R^{(1)} + R^{(1)} \bar{\nabla}^\mu \xi^\sigma - \xi^\sigma \bar{\nabla}^\mu R^{(1)} \right\} \\ - \frac{1}{m^2} \left\{ \xi_\nu \bar{\nabla}^\sigma \mathcal{G}^{(1)\mu\nu} - \xi_\nu \bar{\nabla}^\mu \mathcal{G}^{(1)\sigma\nu} - \mathcal{G}^{(1)\mu\nu} \bar{\nabla}^\sigma \xi_\nu + \mathcal{G}^{(1)\sigma\nu} \bar{\nabla}^\mu \xi_\nu \right\}. \quad (2.24)$$

Thus the conserved charge (2.18) becomes

$$Q(\xi) = -\frac{1}{8\pi G} \int_{\partial M} dS_i \sqrt{-\bar{g}} \mathcal{F}^{0i}. \quad (2.25)$$

Here we choose the spacelike surface as the constant time surface. For asymptotic  $AdS_3$  spacetime, the expression for the conserved charge could be simplified as

$$Q(\xi) = -\lim_{\rho \rightarrow \infty} \frac{1}{8\pi G} \int d\phi \sqrt{-\bar{g}} \mathcal{F}^{0\rho}, \quad (2.26)$$

where  $\rho$  is the radial coordinate of  $AdS_3$ . Note that here to get the formula for the conserved charges, we have used the definition of the Killing vectors  $\bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu = 0$  which does not hold any more for asymptotic symmetries of the spacetime. Thus for asymptotic symmetries which do not obey  $\bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu = 0$ , (2.23) is no longer a conserved quantity and we need to add some terms composed by  $\bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu$  and  $h_{\mu\nu}$  [21]. However, the formula (2.26) will still be valid to the linearized level of gravitational excitations in our consideration.

### 3 Brown-Henneaux boundary condition

In this section we will analyze the Brown-Henneaux boundary condition [45] for the new massive gravity theory. We will calculate the conserved charges corresponding to the generators of the asymptotical symmetry under this boundary condition and see whether all the charges are finite or not. In this and the following sections we will work in the global coordinate system (2.5).

The Brown-Henneaux boundary condition for the linearized gravitational excitations in asymptotical  $AdS_3$  spacetime can be written as

$$\begin{pmatrix} h_{++} = \mathcal{O}(1) & h_{+-} = \mathcal{O}(1) & h_{+\rho} = \mathcal{O}(e^{-2\rho}) \\ h_{-+} = h_{+-} & h_{--} = \mathcal{O}(1) & h_{-\rho} = \mathcal{O}(e^{-2\rho}) \\ h_{\rho+} = h_{+\rho} & h_{\rho-} = h_{-\rho} & h_{\rho\rho} = \mathcal{O}(e^{-2\rho}) \end{pmatrix} \quad (3.1)$$

in the global coordinate system.

The corresponding asymptotic Killing vectors are

$$\begin{aligned} \xi &= \xi^+ \partial_+ + \xi^- \partial_- + \xi^\rho \partial_\rho \\ &= [\epsilon^+(\tau^+) + 2e^{-2\rho} \partial_-^2 \epsilon^-(\tau^-) + \mathcal{O}(e^{-4\rho})] \partial_+ \\ &\quad + [\epsilon^-(\tau^-) + 2e^{-2\rho} \partial_+^2 \epsilon^+(\tau^+) + \mathcal{O}(e^{-4\rho})] \partial_- \\ &\quad - \frac{1}{2} [\partial_+ \epsilon^+(\tau^+) + \partial_- \epsilon^-(\tau^-) + \mathcal{O}(e^{-2\rho})] \partial_\rho. \end{aligned} \quad (3.2)$$

Because  $\phi$  is periodic, we could choose the basis  $\epsilon_m^+ = e^{im\tau^+}$  and  $\epsilon_n^- = e^{in\tau^-}$  and denote the corresponding Killing vectors as  $\xi_m^L$  and  $\xi_n^R$ . The algebra structure of these vectors is

$$i [\xi_m^L, \xi_n^L] = (m-n) \xi_{m+n}^L, \quad i [\xi_m^R, \xi_n^R] = (m-n) \xi_{m+n}^R, \quad [\xi_m^L, \xi_n^R] = 0. \quad (3.3)$$

Thus these asymptotic Killing vectors give two copies of Virasora algebra. To calculate the conserved charges using (2.26) we first parameterize the gravitons as follows

$$\begin{aligned} h_{++} &= f_{++}(\tau, \phi) + \dots \\ h_{+-} &= f_{+-}(\tau, \phi) + \dots \\ h_{+\rho} &= e^{-2\rho} f_{+\rho}(\tau, \phi) + \dots \\ h_{--} &= f_{--}(\tau, \phi) + \dots \\ h_{-\rho} &= e^{-2\rho} f_{-\rho}(\tau, \phi) + \dots \\ h_{\rho\rho} &= e^{-2\rho} f_{\rho\rho}(\tau, \phi) + \dots, \end{aligned} \quad (3.4)$$

where  $f_{\mu\nu}$  depends only on  $\tau$  and  $\phi$  while not on  $\rho$  and the “...” terms are lower order terms which do not contribute to the conserved charges. After plugging (3.4) into (2.26) and performing the  $\rho \rightarrow \infty$  limit, we obtain

$$Q = \frac{1}{8\pi G\ell} \int d\phi \left[ \left(1 - \frac{1}{2m^2\ell^2}\right) (\epsilon^+ f_{++} + \epsilon^- f_{--}) - \left(1 + \frac{1}{2m^2\ell^2}\right) \frac{(\epsilon^+ + \epsilon^-) (16f_{+-} - f_{\rho\rho})}{16} \right] \quad (3.5)$$



for this theory. Three components of the equation of motion (2.10), which do not involve second derivative terms, can be viewed as asymptotic constraints. The  $\rho\rho$  component gives

$$16f_{+-} - f_{\rho\rho} = 0 \quad (3.6)$$

at the boundary and the  $+\rho$  and  $-\rho$  components give

$$(1 - 2m^2\ell^2)\partial_- f_{++} = (1 - 2m^2\ell^2)\partial_+ f_{--} = 0 \quad (3.7)$$

respectively. After plugging in these boundary constraints, the conserved charges become

$$\begin{aligned} Q &= \frac{1}{8\pi G\ell} \int d\phi \left[ \left(1 - \frac{1}{2m^2\ell^2}\right) (\epsilon^+ f_{++} + \epsilon^- f_{--}) \right] \\ &= Q_L + Q_R, \end{aligned} \quad (3.8)$$

where the left moving conserved charge is

$$Q_L = \frac{1}{8\pi G\ell} \int d\phi \left[ \left(1 - \frac{1}{2m^2\ell^2}\right) (\epsilon^+ f_{++}) \right], \quad (3.9)$$

and the right moving conserved charge

$$Q_R = \frac{1}{8\pi G\ell} \int d\phi \left[ \left(1 - \frac{1}{2m^2\ell^2}\right) (\epsilon^- f_{--}) \right]. \quad (3.10)$$

The left moving and right moving conserved charges fulfill two copies of Virasoro algebra with central charges

$$c_L = c_R = \frac{3\ell}{2G} \left(1 - \frac{1}{2m^2\ell^2}\right), \quad (3.11)$$

which are the same with the ones obtained in [5, 28–31]. We can see that the conserved charges  $Q$  are always finite for arbitrary value of  $m$ , so the Brown-Henneaux boundary condition is always consistent with the new massive gravity theory. At the critical point  $m^2\ell^2 = 1/2$ , the conserved charges vanish which provides further evidence to our previous conjecture [5] that the new massive gravity theory may be trivial at the critical point under the Brown-Henneaux boundary condition.

#### 4 Log boundary conditions

At the critical point  $m^2\ell^2 = 1/2$ , new solutions of the equation of motion (2.10) can be constructed following [11] to be

$$\psi_{\mu\nu}^{\text{new}} \equiv \lim_{m^2\ell^2 \rightarrow 1/2} \frac{\psi_{\mu\nu}^M(m^2\ell^2) - \psi_{\mu\nu}^m}{m^2\ell^2 - 1/2}. \quad (4.1)$$

Note here that we have both left and right moving massive and massless modes, so we could obtain both left and right moving new solutions, which are

$$\psi_{\mu\nu}^{\text{newL}} \equiv \lim_{m^2\ell^2 \rightarrow 1/2} \frac{\psi_{\mu\nu}^{ML}(m^2\ell^2) - \psi_{\mu\nu}^{mL}}{m^2\ell^2 - 1/2} = y(\tau, \rho) \psi_{\mu\nu}^{mL} \quad (4.2)$$

and

$$\psi_{\mu\nu}^{\text{newR}} \equiv \lim_{m^2\ell^2 \rightarrow 1/2} \frac{\psi_{\mu\nu}^{MR}(m^2\ell^2) - \psi_{\mu\nu}^{mR}}{m^2\ell^2 - 1/2} = y(\tau, \rho)\psi_{\mu\nu}^{mR} \quad (4.3)$$

respectively. The function

$$y(\tau, \rho) = (-i\tau - \ln \cosh \rho)/2 \quad (4.4)$$

is the same for the two solutions.

These new solutions do not obey the Brown-Henneaux boundary conditions. The energy of these new solutions can be calculated using the Ostrogradsky procedure [8, 11, 46] to be a negative, finite and time-independent value, which is  $-\frac{49}{576G\ell^3}$  for the specific solutions (4.2) and (4.3). This may suggest the instability of the  $AdS_3$  vacuum under the relaxed boundary conditions. However, it may still be stable non-perturbatively and it is still useful to analyze the new boundary conditions.

#### 4.1 Left moving relaxation

In order to include these new interesting solutions, the boundary conditions need to be loosened [22, 25, 26]. Earlier investigations on the relaxation of the boundary conditions for gravity coupled with scalar fields in anti-de Sitter spacetime could be found in [47–51]. Because we have two new solutions, we can relax the boundary condition either to include one of the two solutions or to include both. Thus there are three kinds of boundary conditions. In this subsection we analyze the first kind of these, which is exactly the same as the one used in topological massive gravity for  $\mu > 0$ .

We relax the boundary condition as follows<sup>2</sup> to include the solution  $\psi_{\mu\nu}^{\text{newL}}$ :

$$\begin{pmatrix} h_{++} = \mathcal{O}(\rho) & h_{+-} = \mathcal{O}(1) & h_{+\rho} = \mathcal{O}(\rho e^{-2\rho}) \\ h_{-+} = h_{+-} & h_{--} = \mathcal{O}(1) & h_{-\rho} = \mathcal{O}(e^{-2\rho}) \\ h_{\rho+} = h_{+\rho} & h_{\rho-} = h_{-\rho} & h_{\rho\rho} = \mathcal{O}(e^{-2\rho}) \end{pmatrix}. \quad (4.5)$$

Then the corresponding asymptotic Killing vector can be calculated to be

$$\begin{aligned} \xi &= \xi^+ \partial_+ + \xi^- \partial_- + \xi^\rho \partial_\rho \\ &= [\epsilon^+(\tau^+) + 2e^{-2\rho} \partial_-^2 \epsilon^-(\tau^-) + \mathcal{O}(e^{-4\rho})] \partial_+ \\ &\quad + [\epsilon^-(\tau^-) + 2e^{-2\rho} \partial_+^2 \epsilon^+(\tau^+) + \mathcal{O}(\rho e^{-4\rho})] \partial_- \\ &\quad - \frac{1}{2} [\partial_+ \epsilon^+(\tau^+) + \partial_- \epsilon^-(\tau^-) + \mathcal{O}(e^{-2\rho})] \partial_\rho. \end{aligned} \quad (4.6)$$

Note that these asymptotic Killing vectors are different from (3.2) only in the subleading order, so these also give two copies of Varasoro algebra the same as (3.3).

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<sup>2</sup>This boundary condition is called the log boundary condition in the sense that if we change from the global coordinate to the Poincare coordinate system, the relaxed term is a logarithmic function of the radial coordinate.

With this new boundary condition we can parameterize the asymptotic excitations as follows

$$\begin{aligned}
 h_{++} &= \rho f_{++}^L(\tau, \phi) + \dots \\
 h_{+-} &= f_{+-}^L(\tau, \phi) + \dots \\
 h_{+\rho} &= \rho e^{-2\rho} f_{+\rho}^L(\tau, \phi) + \dots \\
 h_{--} &= f_{--}^L(\tau, \phi) + \dots \\
 h_{-\rho} &= e^{-2\rho} f_{-\rho}^L(\tau, \phi) + \dots \\
 h_{\rho\rho} &= e^{-2\rho} f_{\rho\rho}^L(\tau, \phi) + \dots
 \end{aligned}
 \tag{4.7}$$

Note that  $f_{\mu\nu}^L$  depends only on  $\tau, \phi$  while not on  $\rho$  and the “...” terms are subleading terms which do not contribute to the conserved charge. After plugging (4.7) into (2.26) and performing the  $\rho \rightarrow \infty$  limit, we could obtain

$$Q = \frac{1}{8\pi G\ell} \int d\phi \left[ \left(1 - \frac{1}{2m^2\ell^2}\right) (\infty) - \left(1 + \frac{1}{2m^2\ell^2}\right) \frac{(\epsilon^+ + \epsilon^-)(16f_{+-}^L - f_{\rho\rho}^L)}{16} + \frac{2\epsilon^+ f_{++}^L}{m^2\ell^2} \right].
 \tag{4.8}$$

Here the first term is a linear divergent term proportional to  $\rho$  at infinity, which is caused by the relaxation of the boundary condition. We see that the conserved charges can only be finite at the special point  $m^2\ell^2 = 1/2$ , which means that the log boundary condition is only well-defined at the special point  $m^2\ell^2 = 1/2$ . There are now two asymptotic constraints coming from the equation of motion (2.10), which are

$$16f_{+-}^L - f_{\rho\rho}^L = 0,
 \tag{4.9}$$

and

$$\partial_- f_{++}^L = 0.
 \tag{4.10}$$

Now at the point  $m^2\ell^2 = 1/2$ , the conserved charges become

$$Q_L = \frac{1}{2\pi G\ell} \int d\phi [\epsilon^+ f_{++}^L], \quad Q_R = 0.
 \tag{4.11}$$

It’s interesting that though we have loosened the boundary condition to get nonzero left moving charges, the right moving conserved charges are still zero. The original new massive gravity theory has a left and right moving symmetry and we have symmetric left and right moving modes. However, the “chiral” boundary condition (4.5) breaks this symmetry, which leads to a “chiral” gravity which does not possess the left-right symmetry anymore. Thus the new massive gravity with the boundary condition (4.5) can be viewed as a new kind of log chiral gravity, the chirality of which is realized by imposing “chiral” boundary conditions. Note here that the central charge for the left moving mode is still zero under this boundary condition, similar to [25] for topological massive gravity.

## 4.2 Right moving relaxation

The second kind of boundary condition is similar to the first one, and can be obtained from the first one by exchanging  $\tau^+$  and  $\tau^-$ . This boundary condition is also the same to the log boundary condition of chiral gravity with  $\mu < 0$ .

We relax the boundary condition of the gravitons to be

$$\begin{pmatrix} h_{++} = \mathcal{O}(1) & h_{+-} = \mathcal{O}(1) & h_{+\rho} = \mathcal{O}(e^{-2\rho}) \\ h_{-+} = h_{+-} & h_{--} = \mathcal{O}(\rho) & h_{-\rho} = \mathcal{O}(\rho e^{-2\rho}) \\ h_{\rho+} = h_{+\rho} & h_{\rho-} = h_{-\rho} & h_{\rho\rho} = \mathcal{O}(e^{-2\rho}) \end{pmatrix}. \quad (4.12)$$

Then the corresponding asymptotic Killing vector is

$$\begin{aligned} \xi &= \xi^+ \partial_+ + \xi^- \partial_- + \xi^\rho \partial_\rho \\ &= [\epsilon^+(\tau^+) + 2e^{-2\rho} \partial_-^2 \epsilon^-(\tau^-) + \mathcal{O}(\rho e^{-4\rho})] \partial_+ \\ &\quad + [\epsilon^-(\tau^-) + 2e^{-2\rho} \partial_+^2 \epsilon^+(\tau^+) + \mathcal{O}(e^{-4\rho})] \partial_- \\ &\quad - \frac{1}{2} [\partial_+ \epsilon^+(\tau^+) + \partial_- \epsilon^-(\tau^-) + \mathcal{O}(e^{-2\rho})] \partial_\rho. \end{aligned} \quad (4.13)$$

These asymptotic Killing vectors are different from (3.2) only to the subleading order, and it also gives two copies of Virasoro algebra (3.3).

This time we can parameterize the asymptotic behavior of gravitons as follows

$$\begin{aligned} h_{++} &= f_{++}^R(\tau, \phi) + \dots \\ h_{+-} &= f_{+-}^R(\tau, \phi) + \dots \\ h_{+\rho} &= e^{-2\rho} f_{+\rho}^R(\tau, \phi) + \dots \\ h_{--} &= \rho f_{--}^R(\tau, \phi) + \dots \\ h_{-\rho} &= \rho e^{-2\rho} f_{-\rho}^R(\tau, \phi) + \dots \\ h_{\rho\rho} &= e^{-2\rho} f_{\rho\rho}^R(\tau, \phi) + \dots \end{aligned} \quad (4.14)$$

Note that  $f_{\mu\nu}^R$  depends only on  $\tau, \phi$  while not on  $\rho$  and the “...” terms are lower order terms which do not contribute to the conserved charges. After plugging (4.14) into (2.26) and performing the  $\rho \rightarrow \infty$  limit, we reach

$$Q = \frac{1}{8\pi G\ell} \int d\phi \left[ \left(1 - \frac{1}{2m^2\ell^2}\right) (\infty) - \left(1 + \frac{1}{2m^2\ell^2}\right) \frac{(\epsilon^+ + \epsilon^-)(16f_{+-}^R - f_{\rho\rho}^R)}{16} + \frac{2\epsilon^- f_{--}^R}{m^2\ell^2} \right]. \quad (4.15)$$

The first term is also a linear divergent term and we can see that the conserved charges can only be finite at the critical point  $m^2\ell^2 = 1/2$ , which means that the Log boundary condition is only well-defined for this special point. The two asymptotical constraints from the equation of motion is now

$$16f_{+-}^R - f_{\rho\rho}^R = 0 \quad (4.16)$$

and

$$\partial_+ f_{--}^R = 0. \quad (4.17)$$

Thus at the critical point we have

$$Q_L = 0, \quad Q_R = \frac{1}{2\pi G\ell} \int d\phi [\epsilon^- f_{--}^R]. \quad (4.18)$$

This is also a new chiral gravity with log boundary conditions. The right moving central charge is still zero.

### 4.3 Both modes relaxation

In this subsection we consider the possibility that both  $\psi^{\text{newL}}$  and  $\psi^{\text{newR}}$  can be included after the boundary conditions are relaxed. To reach this, we need to loosen the boundary condition for the gravitational excitations to be

$$\begin{pmatrix} h_{++} = \mathcal{O}(\rho) & h_{+-} = \mathcal{O}(1) & h_{+\rho} = \mathcal{O}(\rho e^{-2\rho}) \\ h_{-+} = h_{+-} & h_{--} = \mathcal{O}(\rho) & h_{-\rho} = \mathcal{O}(\rho e^{-2\rho}) \\ h_{\rho+} = h_{+\rho} & h_{\rho-} = h_{-\rho} & h_{\rho\rho} = \mathcal{O}(e^{-2\rho}) \end{pmatrix}. \quad (4.19)$$

The corresponding asymptotic Killing vector can be calculated to be

$$\begin{aligned} \xi &= \xi^+ \partial_+ + \xi^- \partial_- + \xi^\rho \partial_\rho \\ &= [\epsilon^+(\tau^+) + 2e^{-2\rho} \partial_-^2 \epsilon^-(\tau^-) + \mathcal{O}(\rho e^{-4\rho})] \partial_+ \\ &\quad + [\epsilon^-(\tau^-) + 2e^{-2\rho} \partial_+^2 \epsilon^+(\tau^+) + \mathcal{O}(\rho e^{-4\rho})] \partial_- \\ &\quad - \frac{1}{2} [\partial_+ \epsilon^+(\tau^+) + \partial_- \epsilon^-(\tau^-) + \mathcal{O}(e^{-2\rho})] \partial_\rho. \end{aligned} \quad (4.20)$$

Note that these asymptotic Killing vectors are still different from (3.2) only to the sub-leading order, so it also gives two copies of Virasoro algebra (3.3).

With this boundary condition we can parameterize the asymptotic behaviors of gravitational excitations as follows

$$\begin{aligned} h_{++} &= \rho f_{++}^B(\tau, \phi) + \dots \\ h_{+-} &= f_{+-}^B(\tau, \phi) + \dots \\ h_{+\rho} &= \rho e^{-2\rho} f_{+\rho}^B(\tau, \phi) + \dots \\ h_{--} &= \rho f_{--}^B(\tau, \phi) + \dots \\ h_{-\rho} &= \rho e^{-2\rho} f_{-\rho}^B(\tau, \phi) + \dots \\ h_{\rho\rho} &= e^{-2\rho} f_{\rho\rho}^B(\tau, \phi) + \dots \end{aligned} \quad (4.21)$$

Here  $f_{\mu\nu}^B$  depends only on  $\tau, \phi$  while not on  $\rho$  and the “...” terms are lower order terms which don't contribute to the conserved charges. After plugging (4.21) into (2.26) and performing the  $\rho \rightarrow \infty$  limit, we have

$$Q = \frac{1}{8\pi G\ell} \int d\phi \left[ \left(1 - \frac{1}{2m^2\ell^2}\right) (\infty) - \left(1 + \frac{1}{2m^2\ell^2}\right) \frac{(\epsilon^+ + \epsilon^-)(16f_{+-}^B - f_{\rho\rho}^B)}{16} + \frac{2\epsilon^+ f_{++}^B + 2\epsilon^- f_{--}^B}{m^2\ell^2} \right]. \quad (4.22)$$

The first divergent term is still linear divergent and the conserved charges can only be finite at the special point  $m^2\ell^2 = 1/2$ . This means that this third kind of log boundary condition is also only well-defined at the special point. Different from the previous two kinds of log boundary conditions, now the equation of motions (2.10) gives three asymptotic constraints, which are

$$16f_{+-}^B - f_{\rho\rho}^B = 0 \quad (4.23)$$

and

$$\partial_- f_{++}^B = \partial_+ f_{--}^B = 0 \tag{4.24}$$

respectively.

After drawing terms which vanish under these constraints, we get

$$Q_L = \frac{1}{2\pi G\ell} \int d\phi [\epsilon^+ f_{++}^B], \quad Q_R = \frac{1}{2\pi G\ell} \int d\phi [\epsilon^- f_{--}^B] \tag{4.25}$$

at  $m^2\ell^2 = 1/2$ . This time the left moving and right moving conserved charges are both nonzero. The left and right moving central charges are still zero with the same arguments in [25].

## 5 Conclusion and discussion

In this note we have studied the Brown-Henneaux and log boundary conditions for new massive gravity in asymptotically  $AdS_3$  spacetime. We find that the Brown-Henneaux boundary conditions are always consistent with this theory and at a critical point  $m^2\ell^2 = 1/2$  the conserved charges all vanish and this can be viewed as further evidence that the theory may become trivial at this point under the Brown-Henneaux boundary conditions. Log boundary conditions can also be imposed to this new massive gravity, but it is only consistent at the critical point  $m^2\ell^2 = 1/2$ . According to the boundary conditions of which components of the gravitons are relaxed, we have three kinds of log boundary conditions, and each of them is consistent at the critical point. It is also interesting that the first two kinds of log boundary conditions can give a new kind of log chiral gravity because we have imposed boundary conditions which are not symmetric for the coordinates  $\tau^+$  and  $\tau^-$ .

Although the log boundary conditions for the new massive gravity can be consistent to the linearized level, the physical consistency of log gravity still needs to be studied. It will be interesting to find solutions to the full equation of motion which can be viewed as nonlinear generalizations of the linearized solutions which have the log asymptotic behavior found in this paper, just as what has been done for topological massive gravity in [52, 53]. Also it would be interesting to find other consistent boundary conditions for this new massive gravity in asymptotically  $AdS_3$  at both the critical point and other values of the mass parameter. Further understanding of this new massive gravity theory in asymptotically  $AdS_3$  and the dual field theory would be helpful to the study of quantum gravity in three dimensions.

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